On the Strategy and Profits of Trade-Based Market Manipulation

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This Draft: 11 January 2007

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Abstract

An informed trader can benefit from manipulative trade, just as a card player can benefit from bluffing. Bluffing strategies create ambivalence about the motive for aggressive play and thereby dampen the response of other players. Bluffing itself generally does not garner significant profits because the bluffer holds poor cards. Rather bluffing leads to larger pots and larger profits when a player bets aggressively with a strong hand. Likewise, manipulative trade occurs when an informed trader has a small informational advantage and is not expected to generate excess profits. An informed trader who trades against her information incurs a short-term expected loss and breaks even over the long run, relative to trading with her information. The benefits of manipulative trade are indirect. Manipulative strategies weaken the inferences a market maker can draw from order flow and make price less responsive to order flow. The benefits of decreased price responsiveness are realized when the informed trader has a large informational advantage and doesn't manipulate. Trade-based manipulation is, on average, a zero-cost method of changing market dynamics in favor of informed traders with large informational advantages. In the presence of potential manipulation, prices can be expected to become less efficient throughout the trading process. Manipulation is more effective when informed trade is more visible. Consequently, an informed trader may prefer a less liquid market to a more liquid market. All of our results are shown in a market with a known monopolist informed trader and do not rely on uncertainty about the existence or number of informed traders in the market.

1 Introduction

Stock market manipulation remains an enduring concern for market participants and regulators.[1] Virtually every stock market in the world is subject to statutes that prohibit market manipulation. The perceived dangers of market manipulation include excess price volatility, a lack of price informativeness, and the resulting potential inefficient allocation of capital.

Allen and Gale (1992) categorize market manipulation as falling into three categories: *action-based*, *information-based*, and *trade-based*. In action-based manipulation the manipulator profits by taking actions that change the actual value of the assets. This would include, for example, a manager who takes a short position in his own stock and subsequently undertakes a value-destroying project.[2] In information-based manipulation the manipulator profits by spreading false rumors or releasing false information. Examples of information-based manipulation occurred during the late 1990's internet bubble.[3] In trade-based manipulation the manipulator profits simply by buying and selling a stock without taking any actions to change the fundamental value of the stock nor releasing false information about the value of the stock. This paper studies trade-based manipulation. Allen and Gale (1992) argue that trade-based manipulation is the most difficult type of manipulation to regulate and eradicate. Our findings support this position.

Trade-based manipulation, or 'bluffing,' is said to occur if a trader attempts to influence a security's price by submitting orders that are, at some level, inconsistent with the information she possesses about the security's fundamental value. This can happen two ways. First, an uninformed manipulator can influence prices through trade if other market participants are aware of her trades and incorrectly believe she is informed.[4] Second, an informed trader can

[1] See Sobel (1965) and Twentieth Century Fund (1935). Allen and Gale (1992) provide a brief review.

[2] See Wycoff (1968) for a 1901 incident involving American Steel and Wire Company.

[3] See, for example, United States v. Hoke (PairGain), CR 99-441 (C.D. Cal. indictment filed April 30, 1999).

[4] For example, in "matched orders," someone with little or no information buys (shorts) a thinly traded

trade *against* her information in hopes of recouping her losses in one or more subsequent periods. We demonstrate the feasibility and profitability of this second type of trade-based manipulation.

Our model is a discretized version of Kyle (1985). A single informed trader is endowed with a binary (e.g., bullish or bearish) signal of the liquidation value of a risky asset.[5] The informed trader is known to exist, but her information is private. There are three rounds of trade and in each round the informed trader can submit an order to buy or sell a single share (or not trade). We characterize the informed trader as 'bluffing' or engaging in trade-based manipulation if she buys when she has a bearish signal (and therefore knows the asset is overvalued) or sells when she has a bullish signal (and therefore knows the asset is undervalued). The market also includes liquidity traders and a competitive market maker. The liquidity traders trade for reasons unassociated with the liquidation value of the asset. The aggregate liquidity trade in each round is independent and drawn from a discrete uniform distribution. The competitive market marker sets the market price in each trade round equal to the expected liquidation value of the asset, conditional on the observed aggregate order flow (and all previous trade). The market maker sees only the aggregate order flow. Based on the aggregate order flow, the market maker infers what he can about the informed trader's order flow and the informed trader's information. The model is solved by backward induction.

We solve for the equilibrium in closed-form and show that the informed trader will sometimes

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stock, and subsequently posts heavy buy and sell orders simultaneously in an attempt to create renewed buying (selling) interest in the security. A related strategy involves creating a "run" by aggressively buying or selling a security, in an attempt to attract other buyers (sellers) and push up (down) the price.

[5]Our assumption of a single informed trader is important, but need not be interpreted literally. Even if there exists multiple informed traders, each is likely to possess some degree of unique information. In this sense, the model may be viewed as studying the marginal component of an informed trader's order, that which is orthogonal to the information-based trade of other informed traders. Section 4 discusses the impact of multiple informed traders in more detail. Holden and Subrahmanyam (1992) study a multi-period Kyle (1985) model with multiple identically informed insiders. Foster and Viswanathan (1996) study a multi-period Kyle model with multiple differently informed insiders. Callahan (2004) studies a multi-period Kyle-type model with an unknown number of identically informed insiders. Dridi and Germain (2004) study a one period Kyle-type model with multiple identically informed insiders with binary signals.

bluff and trade against her information. We examine how the likelihood of bluffing varies as a function of the trading horizon, the degree of informational advantage, and the ability of the market maker to accurately infer the informed order flow from the aggregate order flow. We examine the impact of bluffing on the expected profits of the informed trader and on market price dynamics, like liquidity and price efficiency.

Consideration of the strategic interaction between an informed trader's behavior and the market pricing function is not new. Kyle (1985) demonstrates that an informed trader optimally balances the benefits and costs associated with trading aggressively. On the one hand, submitting large trades yields higher profits (all else equal), but on the other hand, submitting large trades moves prices more quickly toward fundamental value. This tradeoff is also present in our model, but - departing from Kyle (1985) - we show that the informed trader not only modulates the intensity of her trade, but in addition sometimes trades *against* her information. Therefore, in our model it is not simply a matter of how quickly an informed trader pushes price toward fundamentals, but also in what circumstances might an informed trader push price *away* from fundamentals and lessen market efficiency.

Our most important result concerns the additional profits associated with bluffing. Intuitively, an informed trader will trade against her information only when her informational advantage over the market is small, since the opportunity cost of foregone gains is equally small. That is, manipulation is more likely when prices are relatively efficient and close to fundamental value. Yet, the increase in expected profits from manipulation occurs when prices are *far* from fundamentals. It follows that in general an informed trader will not increase her expected profits as a direct consequence of price manipulation. In fact, we show that when prices are close to fundamental value, the informed trader is more likely to trade against her information, yet her future expected profits are no greater than she would earn by adopting a non-manipulative trade strategy. In contrast, when prices are far from fundamentals, the informed trader does not manipulate price and trades with her information, yet her future expected profits are signficantly higher than she would earn in a market without the possibility of manipulative trade. The benefits of manipulation are thus indirect: manipulation is a zero-cost method of changing the market dynamics to favor informed traders. Our findings suggest that it will be very difficult to find evidence of trade-based manipulation if such evidence requires both trade reversals and excess profits. In general, informed traders with small informational advantages will engage in manipulative behavior but not earn excess profits, while informed traders with large informational advantages will not engage in manipulative behavior but will earn excess profits.

To better understand this, consider how bluffing helps a card player in a game of poker. For a given hand of cards, a player can bet so as to maximize his profits in that round. That is, if he has good cards he bets aggressively and vice versa. If the other players understand his strategy however, they are unlikely to challenge aggressive betting by matching or raising the pot,[6] making it difficult for someone with a good hand to profit from it. Ideally, a card player dealt a good hand would like to bet aggressively without alerting the table's other players of his quality hand. Card players have long recognized this problem and its solution - bluffing. Sometimes a player with *poor* cards bets aggressively (bluffs), potentially sustaining a loss if the poor cards are revealed at the game's completion. Like manipulation in our model, bluffing is likely to occur with poor cards (a small informational advantage), but is not expected to yield immediate profits from the bluff itself. Indeed, the benefits of bluffing arise when the player has good cards. In this case the player can bet aggressively without revealing the quality of his hand. The other players know that aggressive bets don't necessarily signal a good hand and they become less responsive to the aggressive play. This allows the player with a good hand to raise the pot without causing the other players to fold. Likewise in a market where trade-based manipulation is known to occur under some circumstances, the market is deeper and price is less sensitive to order flow. The benefits to the informed trader of increased market depth are realized at times when the informed

[6] That is, unless someone thinks he has better cards himself.

trader does not engage in manipulation (though an informed trader with different information might).

A second class of implications speaks broadly to the impacts of manipulation on market stability and efficiency. Manipulation is destabilizing in that it moves prices away from fundamentals. Our findings suggest that this effect may be self-reinforcing. Previous research has shown that if liquidity traders have discretion over the timing or location of their trades, they will tend to avoid trading where or when liquidity is low or there exists a high proportion of informed traders.[7] This is because informed trader profits are financed by liquidity trader losses. Therefore, factors that increase informed trader profits will increase liquidy trader losses and tend to drive liquidity traders toward alternative trading venues.

We find that manipulation is more likely and more profitable in settings with higher informational trade transparency - that is, in markets where participants are better able to distinguish informed trade from non-informed trade. In general, this should tend to include markets with low trade volume, markets with a high proportion of informed traders, and markets with lower trader anonymity. Therefore, market characteristics that are favorable to manipulative trade correspond with market characteristics that liquidity traders try to avoid. Consequently, if a market suffers a liquidity or information shock that increases the likelihood of manipulation, liquidity traders may subsequently withdraw from the market, further increasing the likelihood of manipulative trade and further destabalizing the market. Although we do not endogenize liquidity trader choice, it is a natural extension of the model that fits well with previous literature exploring the dynamics of liquidity trade.[8]

Our finding that informed manipulation is more likely when the market maker is better able to infer the informed order flow is consistent with prior work showing that mandatory

[7] See, e.g., Admati and Pfleiderer (1988) and Chowdhry and Nanda (1991).

[8] Importantly, Spiegel and Subrahmanyam (1992) show that inferences drawn from models with exogenous liquidity trade may not hold up if liquidity traders are replaced with rational maximizers trading to satisfy hedging demands.

disclosure laws can increase the incentives for informed traders to engage in price manipulation (e.g., John and Narayanan (1997)). The probablistic (versus mandatory) nature of disclosure in our model, however, provides additional insight by allowing us to characterize the incentives to manipulate when informed trading is not perfectly revealed. As such, we characterize the manipulation incentives for *all* informed traders, not simply those subject to mandatory disclosure for whom, as we show, manipulation incentives may be the most severe. In addition, we study the incentives to manipulate apart from those generated through regulatory statutes (e.g., mandatory disclosure), so that the set of markets to which our model applies is potentially expanded. Although less regulated markets may lack statutes mandating disclosure of informed trade, we show that these markets may still exhibit trade-based manipulation.

Indeed, we find that informed traders may benefit from increasing the transparency of their trading activities. For example, large hedge funds and investment companies often break up large trades to mitigate the associated price impact. To the extent that these price impacts are associated with adverse selection, our paper suggests that such practices are inconsistent with manipulation. Recuperative gains from manipulation are bigger when the price impact of the manipulative trade is large. Therefore, relatively more transparent and visible informed trade is more consistent with the pursuit of a manipulative trading strategy.

Also of interest is the interaction between manipulation and information production. An informed trader is more likely to manipulate when the informational gap between the informed trader and market maker is narrow and prices more accurately reflect fundamental values. This suggests that the relationship between information production and price efficiency isn't necessarily straightforward. When more information is produced (due, for example, to an increased number of analysts following a stock), prices will tend to be closer to fundamental value, all else equal. However, because prices are closer to fundamental value all else is not equal: informed traders have a greater incentive to manipulate and push prices away from fundamental value. Since manipulation is more likely in earlier periods when losses (or foregone gains) can be recovered, early information production may actually decrease *average* price efficiency.

The rest of the paper proceeds as follows. Section 2 discusses our paper in the context of related literature. Section 3 describes the model and it's solution. Section 4 discusses the interpretation and implications.

2 Related Literature

Our model is based on Kyle (1985), but differs in several ways that allow us to study trade-based manipulation. In Kyle (1985) the liquidation value of the risky asset and noise trader order flow are normally distributed and the insider order can be of any size. These features provide the tractibility necessary to solve for the unique linear equilibrium. Kyle does not consider non-linear equilibria or mixed trading strategies for the insider. Manipulative strategies are expected to be non-linear and/or involve mixed trading strategies for the informed trader. Accordingly, our model is designed to be tractable with respect to non-linear and mixed strategy equilibria.[9] Kyle (1985) shows, and Back (1992) proves formally, that in the continuous time limit market liquidity is constant over the trading horizon. The constant market depth makes manipulation unprofitable for the informed trader. With multiple discrete auctions, however, market liquidity is not constant. This suggests that one can not necessarily preclude the possibility that manipulative trade may be profitable in the multi-period discrete-time Kyle model. While we cannot say whether manipulative trade is or is not profitable in a multi-period discrete-time Kyle model, in sections 3 and 4 we provide insight into the possibility by discussing in more detail the distinguishing features of our

[9] As shown in Section 3, manipulation is never optimal in a one-period game, therefore the restriction to linear equilibria is only relevant for the muli-period Kyle model.

model that allow us to solve for an equilibrium involving manipulative trade.

Our paper is related to existing work in the area of trade-based market manipulation. Chakraborty and Yilmaz (2004) is similar to our paper in several ways. Like us, they study a Kyle (1985) type setting with finite discrete order flow and liquidation value. They prove that in equilibrium an informed trader will always manipulate so long as (i) his information is sufficiently long-lived, and (ii) the market maker is uncertain as to the existence of the informed trader. This second feature is key to their result because "In effect, the informed trader exploits the market makers' uncertainty about his presence to create his own liquidity or noise."[10] We show that even an informed trader who is known with certainty to exist still has incentives to manipulate in some circumstances. Our paper also differs from Chakraborty and Yilmaz in that we provide a constructive proof of a specific manipulative equilibrium whereas their focus is on proving the existence of manipulative equilibria in general. Characterizing a specific equilibrium gives us further insight into the incentives and dynamics of manipulative trading strategies and the associated profits. We examine the size and nature of the expected gains from manipulation (relative to a non-manipulative strategy) and characterize the circumstances under which the expected gains are actually realized.

Other closely related papers include Allen and Gale (1992), Fishman and Hagerty (1995) and John and Narayanan (1997). In Allen and Gale (1992) and Fishman and Hagerty (1995) trade-based manipulation is sometimes profitable for an uninformed trader. In these models, informed traders do not manipulate. The uncertain presence of informed traders in the market, however, is crucial to their results since, as in Chakraborty and Yilmaz (2004), uncertainty about whether or not traders are informed drives the results.

John and Narayanan (1997) extend the Fishman and Hagerty (1995) model and show that

[10] Chakraborty and Yilmaz (2004), page 189.

even a known informed trader may choose to manipulate the market. Their model differs from ours in at least two significant ways. First, the market maker in John and Narayanan (1997) transacts trades in each round at preset bid and ask prices, independent of order volume.[11] In our model the market maker transacts trades at a price that is set conditional on the order size. The significance is that, conditional on contemporaneous trade volume, the transaction prices in John and Narayanan (1997) are not necessarily regret-free for the market maker. Because market prices are conditionally less efficient in John and Narayanan (1997) than in our model, their results may overstate the profitability of manipulative strategies.

The second difference worth noting is that John and Narayanan (1997) impose mandatory disclosure of informed trades (as do the other models under discussion). This suits well their stated purpose of exploring the impact of disclosure laws on the trading incentives of informed insiders. Our interest is in studying the incentives to manipulate for general informed traders, not necessarily insiders subject to disclosure laws. Therefore, in our model there is a non-zero chance the market maker will be able to perfectly infer the informed order flow, but doing so is not guaranteed.

In sum, previous research on trade-based manipulation has shown two conditions that can be sufficient to induce manipulation: (i) mandatory disclosure of insider trades, and (ii) uncertainty about whether or not a known trader is informed. We show that while either of these conditions may be sufficient, neither is wholly necessary. We demonstrate trade-based manipulation in a market without mandatory disclosure in which a monopolist informed trader is known to exist. More importantly, we provide a constructive proof of a manipulative equilibrium and study the nature and dynamics of the manipulative strategy and market pricing function.

[11] The market structure in John and Narayanan (1997) is the same as that in Admati and Pfleiderer (1989), Easley and O'Hara (1992) and Fishman and Hagerty (1995).

3 Model

3.1 Economic Environment

A single risky asset is traded in a market with three types of agents: a single risk-neutral informed trader, a competitive market maker, and noise traders. There are three successive rounds of trade. The asset pays a single cash flow $\tilde{v}$ after the final round of trade. Prior to trade $E[\tilde{v}] = p_0$. For simplicity, the discount rate between successive trade rounds is assumed to be zero. Prior to the market opening for trade an informed trader receives a binary signal $s \in \{l, h\}$ that is perfectly correlated with the asset payoff. Without loss of generality, we set $E[\tilde{v}|l] = 0$ and $E[\tilde{v}|h] = 1$. In each round of trade the informed trader can buy one share, sell one share, or not trade (i.e., sit out of the market). The informed trader's order flow in round n is denoted x_n. Therefore, $x_n \in \{-1, 0, +1\}$ for $n = 1, 2, 3$. x_n may be the outcome of a mixed trading strategy. We denote the informed trader's trading strategy in round n as $X_n(s; p_{n-1})$. The per trade round order flow from noise traders, denoted u_n, is i.i.d. discrete uniform $[-w, +w]$.[12] A competitive market maker observes the aggregate order flow in each round and sets price equal to the expected value of the asset. The aggregate order flow is denoted $z_n = x_n + u_n$ for $n = 1, 2, 3$ and the market price set by the market maker in each round is denoted p_n. We denote the market maker's pricing function in round n as $P_n(z_n; p_{n-1})$.[13]

This setting is the same as that of Kyle (1985), but with different distributional assumptions and a restriction on the informed trader's order size. Specifically, the informed trader's information is binary rather than continuous, the noise trader order flow is discrete uniform

[12] All the results presented hold for $w > 3$. Some results need to be modified for $w \leq 3$. We don't present detailed results for $w \leq 3$.

[13] The informed trading strategy and pricing function are more properly denoted as $X_n(s, p_{n-1}, \ldots, p_0)$ and $P_n(z_n, \ldots, z_1, p_0)$. However, market efficiency dictates that prices follow a martingale which justifies the notation used in the text.

rather than normal, and the informed trader is restricted to buy or sell a single share (or sit out) rather than submit orders of arbitrary size. We do not argue that our assumptions are better (or worse) than those of Kyle. Simply, our modeling choices provide a tractible framework within which we can consider both linear and nonlinear equilibria. Manipulative trade strategies are by their very nature non-linear. The framework of Kyle allows for the solution of linear equilibria, but nonlinear equilibria are intractable and therefore neither ruled out nor confirmed.[14] Overall, our assumptions equate to a discretization of the model. With a discrete-space model we can explore and solve for all equilibria. The discretization is the key departure; the specific discrete distributions chosen are less consequential. The model would be more tedious to solve, for example, if the informed trader were permitted to submit orders ranging from $-k$ to $+k$ shares, but the qualitative nature of the results would remain. Similarly, if the informed trader's information were, e.g., binomial rather than binary, the qualitative nature of our results would not change. We continue our discussion of the implications of our distributional assumptions in Section 4.

3.2 Definition of Equilibrium

An equilibrium for the model comprises an informed trader trade strategy, $X = (X_1, X_2, X_3)$, and a market maker pricing function, $P = (P_1, P_2, P_3)$ such that the informed trader maximizes her expected future profits:

$$\sum_{m=n}^{3} E\left[(\tilde{v} - p_m)x_m(X, P)|s, p_0, \ldots, p_{m-1}\right] \geq \sum_{m=n}^{3} E\left[(\tilde{v} - p_m)x_m(X^*, P)|s, p_0, \ldots, p_{m-1}\right] \quad \forall\ X^* \neq X \text{ and } n = 1, 2, 3$$

[14]Nonlinear equilibria do not exist in a one-period Kyle model. In addition, Back (1992) proves that nonlinear equilibria do not exist in the continuous time Kyle setting.

and price equals the expected future asset payoff conditional on the observed order flow:

$$p_n = E[\tilde{v}|p_0, z_1, \ldots, z_{n-1}] \text{ for } n = 1, 2, 3.$$

3.3 Optimal Strategies

The model is solved by backward induction. The equilibrium is presented and discussed from the perspective of an informed trader with a high signal ($s = h$). Given this perspective, when the informed trader buys a share she is trading with her information and when an informed trader sells a share she is trading against her information. Throughout the paper, we define trade-based market manipulation as trading against one's information, so that sitting out of the market and not trading is not construed as market manipulation. The following proposition presents an equilibrium to the 3-period model, a detailed proof of which is contained in the appendix.[15]

Proposition 1 *In the first round of trade, the informed trader's trading strategy $X_1(h; p_0)$ is a mixed strategy that depends on the intial price of the risky asset p_0. Specifically,*

$$X_1(h; p_0) = \begin{cases} -1 & \text{w.p.} \quad \phi_1^h(p_0) \\ 0 & \text{w.p.} \quad \theta_1^h(p_0) \\ +1 & \text{w.p.} \quad 1 - \phi_1^h(p_0) - \theta_1^h(p_0), \end{cases}$$

where the functional forms for $\phi_1^h(p_0)$ and $\theta_1^h(p_0)$ are given in the appendix. There exists a non-empty set of prices $p^{C3} < p_0 \leq 1$ for which $\phi_1^h(p_0) > 0$, i.e., for some prices the informed trader trades against her information with strictly positive probability. There exists a larger

[15] We prove existence, but not uniqueness, of the equilibrium. There exist, at least, additional equilibria that differ from the presented equilibrium in ways that are economically insignificant. For example, if the informed trader's information is fully reflected in market price prior to the last round of trade (as can happen), in later rounds the informed trader is indifferent between all feasible trading strategies as each and every one has zero expected profits.

set of prices $p^{C1} < p_0 \leq 1$ *where* $p^{C1} < p^{C3}$ *for which* $\theta_1^h(p_0) > 0$*, i.e., the informed trader does not trade with some probability during the first round.*

In the second round of trade, the informed trader's trading strategy $X_2(h; p_1)$ *is a mixed strategy that depends on the first period price of the risky asset* p_1*. Specifically,*

$$X_2(h; p_1) = \begin{cases} 0 & \text{w.p.} \quad \theta_2^h(p_1) \\ +1 & \text{w.p.} \quad 1 - \theta_2^h(p_1), \end{cases}$$

where the functional form of $\theta_2^h(p_1)$ *is given in the appendix. There exists a non-empty set of prices for which the informed trader will sit out during the second round of trading.*

In the third and final round of trade, the informed trader's trading strategy is the following pure strategy:

$$X_3(h; p_2) = 1$$

In all trading rounds $n = 1, 2, 3$*, the market maker sets prices equal to the expected liquidation value of the asset, given the insider's trading strategy and total order flow:*

$$P_n(z_n; p_{n-1}) = \begin{cases} \frac{\phi_n^h(p_{n-1}) \cdot p_{n-1}}{\phi_n^h(p_{n-1}) \cdot p_n + [1 - \theta_n^h(p_{n-1}) - \phi_n^h(p_{n-1})](1 - p_{n-1})} & \text{for} \quad z_n = -w - 1 \\ \frac{[\theta_n^h(p_{n-1}) + \phi_n^h(p_{n-1})]p_{n-1}}{[\theta_n^h(p_{n-1}) + \phi_n^h(p_{n-1})]p_{n-1} + [1 - \phi_n^h(1 - p_{n-1})](1 - p_{n-1})} & \text{for} \quad z_n = -w \\ p_{n-1} & \text{for} \quad -w + 1 \leq z_n \leq w - 1 \\ \frac{[1 - \phi_n^h(p_{n-1})]p_{n-1}}{[1 - \phi_n^h(p_{n-1})]p_{n-1} + [\theta_n^h(1 - p_{n-1}) + \phi_n^h(1 - p_{n-1})](1 - p_{n-1})} & \text{for} \quad z_n = w \\ \frac{[1 - \theta_n^h(p_{n-1}) - \phi_n^h(p_{n-1})]p_{n-1}}{[1 - \theta_n^h(p_{n-1}) - \phi_n^h(p_{n-1})]p_{n-1} + \phi_n^h(1 - p_{n-1}) \cdot (1 - p_{n-1})} & \text{else.} \end{cases}$$

4 Discussion

4.1 Optimal Strategies

During each round, the informed trader's optimal strategy is determined by comparing the relative costs and benefits associated with each of the available pure strategies. In the final round of trading, this comparison is straightforward: trading with one's information delivers a positive expected profit,[16] sitting out of the market and not trading on one's information results in zero profit, and trading against one's information yields an expected loss. Because there are no future rounds of trade in which to recoup a loss or otherwise benefit from not trading with one's information, it follows that in the last round of trade the informed trader will always trade with her information.

Her behavior differs in the second to last round. For some prices, the informed trader will follow a mixed strategy in which she sometimes trades with her information and sometimes does not trade. The informed trader will never trade against her information in the second to last period. To understand why an informed trader may not trade on her information, consider the costs and benefits of deviating away from a pure strategy in which the informed trader always trades with her information.

If the informed trader always trades with her information then the price the market maker sets after observing the aggregate order flow can take only three values. If aggregate order flow is sufficiently low, $z \in \{-w-1, -w\}$, the market maker knows the informed trader sold and sets $p_2 = 0$. Conversely, if order flow is sufficiently high, $z \in \{w+1, w\}$, the market maker knows the informed trader bought and sets $p_2 = 1$. Otherwise, the order flow contains no information about the direction of informed trade and the market maker

[16] The single exception is when $p_2 = 1$, in which case the informed trader's expected profits are zero.

maintains $p_2 = p_1$.[17] Therefore, with probability $\frac{2}{2w+1}$ the informed trader's information is revealed and she earns no current nor future profits. With probability $\frac{2w-1}{2w+1}$ however, $p_2 = p_1$, giving an informed trader profit of $(1 - p_1)$ in the current period and an expected profit of $\frac{2w-1}{2w+1}(1-p_1)$ in the final period. For an informed trader with a high signal, the pure strategy of always trading with one's information yields an expected profit in the final two rounds of:

$$\overbrace{\left(\frac{2w-1}{2w+1}\right)(1-p_1)}^{\textit{second to last period}} + \underbrace{\left(\frac{2w-1}{2w+1}\right)^2 (1-p_1)}_{\textit{last period}}. \tag{1}$$

Does the informed trader have an incentive to deviate from the pure strategy? Yes.

If the informed trader chooses not to trade during the second to last round she will forfeit her expected profits in that round (the first term in the above expression). In return, she lowers the expected price at which she will transact in the final round. Specifically, by not trading during the second to last period, the informed trader generates the following price distribution:[18] (1) $p_2 = \mathrm{E}[v|s]$ w.p. $\frac{1}{2w+1}$ (since $z = \pm(w+1)$ is now impossible), (2) $p_2 = p_1$ w.p. $\frac{2w-1}{2w+1}$ as before, and (3) $p_2 = 1 - \mathrm{E}[v|s]$ w.p. $\frac{1}{2w+1}$. By not trading, there is now only a $\frac{1}{2w+1}$ chance that the market maker correctly infers the informed trader's information and there is an equal chance that the market maker incorrectly infers the informed trader's information (and sets the price completely wrong). Expected informed trader profits for the last period become

[17]For this range of order flows, $-w+2 \leq z \leq w-2$, the market maker's posterior belief (that is, after seeing the aggregate order flow) about the nature of the informed trader's signal is unchanged. Consider, for example, if $z = -w+2$. Since the proposed equilibrium prescribes that the informed trader trades with his information, then $z = -w+2$ would have arisen from either $\{x = +1$ and $u = -w+1\}$ or $\{x = -1$ and $u = -w+3\}$. These events arise with probability p_1 and $1-p_1$ respectively, the market maker's beliefs prior to seeing the aggregate order flow.

[18]Here we are holding the market maker's pricing function fixed. Of course, in equilibrium the market maker's pricing function will adapt to the mixed strategy of the informed trader. For now, we are simply demonstrating that the informed trader has an incentive to deviate from the pure strategy equilibrium.

$$\left(\frac{2w-1}{2w+1}\right)\left(\frac{1}{2w+1}\right)+\left(\frac{2w-1}{2w+1}\right)^2(1-p_1), \tag{2}$$

where the first term is the benefit of having "tricked" the market maker by sitting out the market and not trading.[19]

Only the first terms of expressions (1) and (2) differ. The first term in (2) is positive and independent of price while the first term in (1) decreases in price. Therefore, it can *never* be an equilibrium for the informed trader to trade with his information for all prices. When price is sufficiently high, i.e., close to fundamental value, the informed trader has an incentive to not trade on her information and allow the price to 'drift' away from fundamentals prior to the final round of trade. An informed trader with a larger informational advantage, however, will follow the pure strategy of always trading with her information because the foregone profits from sitting out are too large relative to the benefit of a possibly less efficient price in the last round of trade.

Sitting out the market and not trading on one's information, of course, is not manipulation. Sitting out the market is at least partially an artifact of having a discrete order size: there are prices for which the informed trader would prefer to trade a small fraction of a share rather than none at all. Nevertheless, the above intuition is very useful in understanding if, when, and why an informed trader will manipulate and trade against her information. Manipulation only happens in the first of three rounds of trade. In this case, the marginal benefit of trading against one's information is that the market maker may move price *away* from fundamental value, increasing future expected profits. The marginal cost is the difference in the current round between the expected profit from trading with one's information and the expected

[19]With probability $\frac{1}{2w+1}$, the second to last period's aggregate order flow is exactly $-w$, which causes the market maker to set $p_2 = 1 - \text{E}[v|h] = 0$. In the last period the informed trader will buy. With probability $\frac{2w-1}{2w+1}$ the aggregate order flow will not reveal the informed trader's order, giving her a profit of 1 in the last period. Combining these independent probabilities gives the first term of (2).

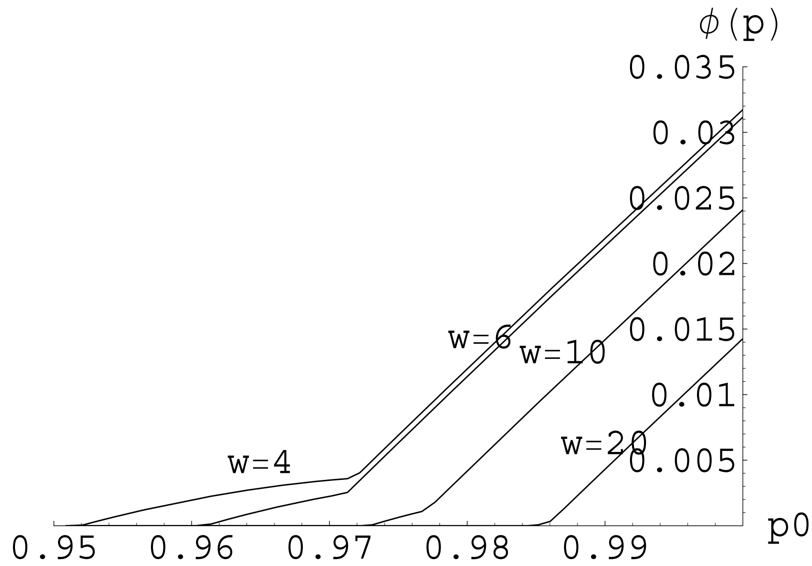

Figure 1: w-contours of $\phi(p_0)$ - Probability of Manipulative Trade

loss from trading against one's information. Like the 2-period case described in detail above, the marginal benefit is relatively constant in price while the marginal cost decreases in price (when the fundamental value is high). When prices are far from fundamentals therefore, it is never worthwhile to trade against one's information. When prices are close to fundamentals, the marginal benefit of manipulating can be made equal to the marginal cost by choosing the appropriate mixing probabilities of each strategy. The probability of trading against one's information increases in price. This probability, $\phi(p_0)$, is shown in Figure 1.

As the initial price, p_0, moves from 0 to 1, an informed trader with a high signal initially adopts a pure strategy of always trading with his information. When the price gets sufficiently high however, the informed trader begins to mix between trading with his information and not trading on his information (this probability, $\theta(p_0)$ is shown in Figure 2). As the initial price becomes higher still the informed trader mixes between all three elements of his strategy space: trading with his information, not trading on his information, and trading against his information. As shown in Figure 1, the likelihood of trading against his information increases with price. The likelihood of sitting out increases in price until the point

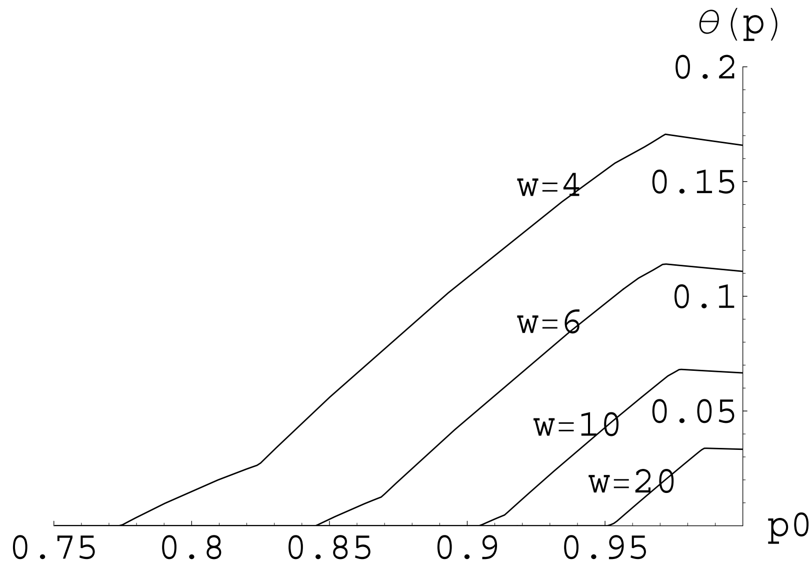

Figure 2: w-contours of $\theta(p_0)$ - Probability of No Trade

when the informed trader begins to manipulate by trading against his information, at which point the likelihood of not trading on his information falls.

The *combined* probability of not trading on one's information, or trading against one's information, increases in price. The combined probability, $(\theta + \phi)$, is shown in Figure 3. Notice that these probilities decrease in w. In this setting, one can interpret w, the noise trade distribution parameter, in two ways. First, w is a measure of the amount of noise in the market during each trade round. Second, w is an inverse measure of the likelihood that the informed trader's order will be perfectly inferable from the aggregate order flow. This second interpretation is of particular interest because we are considering a setting in which the informed trader is not subject to mandatory disclosure. That is, the informed trader is not assumed to be an "insider." This interpretation of (the inverse of) w corresponds to what we refer to as *informational trade transparency*, namely the likelihood that informationally motivated trades are recognized as such.

A market with high informational trade transparency is able to extract more information

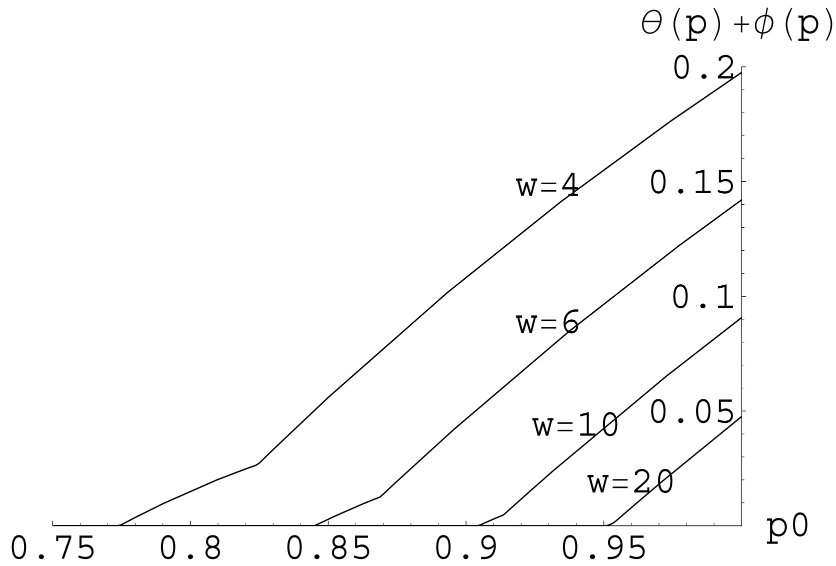

Figure 3: w-contours of $\theta(p_0) + \phi(p_0)$

from the order flow. For example, consider a market in which the market maker sees individual orders. This market would have a very high informational trade transparency if all informed traders had no non-information based motives for trade (e.g., liquidity) and were subject to mandatory disclosure rules. If informed traders had liquidity motives for trade in addition to their information-based motives, the market would have a lower informational trade transparency. Finally, if the informed traders were not required to disclose their trades the market would have an even lower informational trade transparency. In a market with no informational trade transparency no information could be inferred from the order flow. In our model, a large w proxies for a market with low informational trade transparency and a small w proxies for a market with high informational trade transparency.

Informed traders clearly prefer markets with lower informational trade transparency. In such markets informed traders can transact undetected and earn large profits. In such markets manipulation is also less likely. The benefit of trading against one's information is in moving prices away from fundamentals to increase future expected profits. In markets with low informational trade transparency the likelihood of an informed trader's order moving prices is lessened, so the incentives to manipulate are reduced. This raises an interesting tension. On the one hand, higher informational trade transparency increases market efficiency via a more direct link between informed order flow and market price adjustments. On the other hand, higher informational trade transparency increases the incentives for informed traders to manipulate prices by trading against their information such that the "informed" order flow becomes less informative. This point has been discussed by Fishman and Hagerty (1985) as it pertains to mandatory disclosure laws. We demonstrate that this is a general consideration that pertains to any aspect of the market mechanism that impacts informational trade transparency.

4.2 Informed Trader Profits

We now present a corollary to Proposition 1 that quantifies the expected profits of an informed trader with a high signal of the risky asset's value. Expected profits conditional on a low signal are symmetric.

Corollary 1 *The informed trader's expected trading profits for the 3-period game are a decreasing, piecewise continous, and linear function in price. Below are the expected profits for an insider receiving the high signal ($s = h$) prior to the first period's trading activity.*

$$E(\pi|p_0) = \begin{cases} \pi_{base} = \frac{(1-p_0)(2w-1)(1+12w^2)}{(1+2w)^3}, & \text{if } 1 \geq p_0 \geq 1 - p_0^{C4} \\ \pi_a = \pi_{base} + \frac{(1+12w^2)(8w^2-p_0(-1+2w)(1+2w)^2)}{16w^2(1+2w)^3}, & \text{if } 1 - p_0^{C4} \geq p_0 \geq 1 - p_0^{C3} \\ \pi_b = \pi_a + \frac{(1+12w^2)(8w^2-p_0(1+2w)^3)}{16w^2(1+2w)^3}, & \text{if } 1 - p_0^{C3} \geq p_0 \geq \frac{1}{1+2w} \\ \pi_c = \pi_b + \frac{4w(-1+2w)(1-p_0-2p_0w)}{(1+2w)^3}, & \text{if } \frac{1}{1+2w} \geq p_0 \geq 1 - p_0^{C2} \\ \pi_d = \pi_c + \frac{(1+12w^2)(4w(-1-4w+4w^2)-p_0(1+2w)^2(1+12w^2))}{4w(1+2w)^3(-1-4w+12w^2)}, & \text{if } 1 - p_0^{C2} \geq p_0 \geq 1 - p_0^{C1} \\ \pi_e = \pi_d + \frac{(1+12w^2)(8w^2(-1-8w+4w^2)-p_0(1+2w)^3(1+12w^2))}{(1+2w)^3(1+12w+16w^2-112w^3+48w^4)}, & \text{if } 1 - p_0^{C1} \geq p_0 \geq 0. \end{cases}$$

The expressions for the price region boundaries p_0^{C1}, p_0^{C2}, p_0^{C3}, and p_0^{C4} are given in the

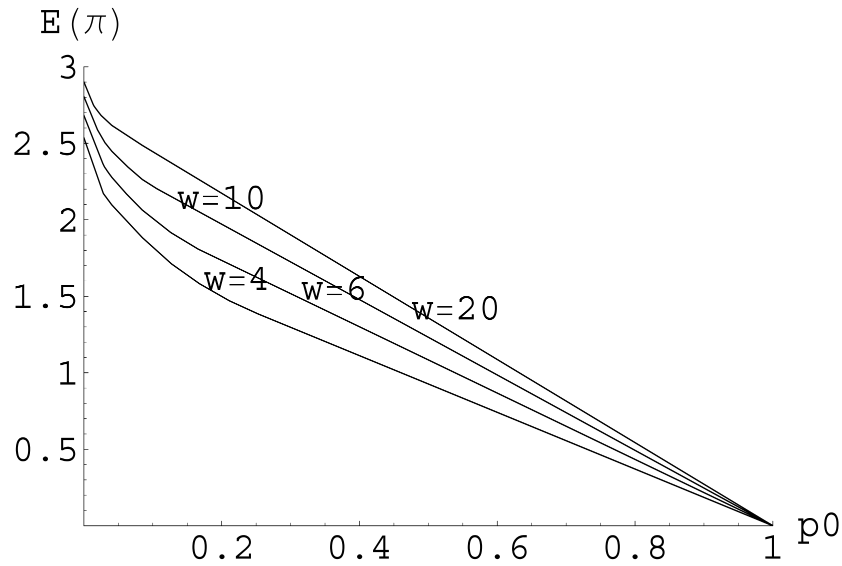

Figure 4: w-contours of $E(\pi(p_0))$

appendix.

Figure 4 plots the expected 3-period profits of the informed trader as a function of pre-trade price, p_0, for various values of w. Larger values of w correspond to a market that is more liquid with a lower degree of informational trade transparency. Because liquidity increases with w, profits also increase with w. More interesting is the shape of the expected profit curves for each w. The curves are drawn for an informed trader with a high signal. Expected profits decrease in p_0 as expected: informed trader profits are lower on average when she has a smaller informational advantage. What is striking is the region in which expected profits are elevated (relative to a non-manipulation benchmark). Recall that an informed trader with a high signal may manipulate when price is close to fundamental value, but does not manipulate when price is far from fundamental value. In contrast, Figure 4 shows that the informed trader earns excess profits when price is far from fundamental value, and not when price is close to fundamental value. That is, in price regions where the informed trader engages in trade-based manipulation, her expected profits simply match those she would earn from not manipulating and always trading with her information. While in price regions where the informed trader exclusively trades *with* her information, she earns excess expected profits. This means that the high-type informed trader earns excess profits in the price region where manipulation would occur if a low-type informed trader were in the market and the low-type informed trader earns excess profits in the price region where manipulation would occur if a high-type trader were in the market.

Consider the case when price is close to zero. A price close to zero indicates that the market maker believes there is a relatively high probability that a low-type informed trader is in the market. The market maker also recognizes that when prices are close to zero, a low-type informed trader may trade against her information and submit a buy order. Therefore, if the market maker infers that an informed trader submitted a buy order, the market maker updates his beliefs based on the relative likelihood that the order came from a low-type informed trader trading against her information versus from a high-type informed trader trading with her information. Because the market maker has a high prior that the informed trader has a low signal, the market maker is reluctant to raise price too much even when he is certain that the informed trader submitted a buy order. This is an ideal situation for a high-type informed trader. Like a card player who has bluffed in the past when her cards were poor but now has a good hand, she can trade with her information and not cause the price to move too far toward fundamental value even when the market maker perfectly infers the informed trader order flow. Therefore, the expected profits for a high-type informed trader are elevated due to the likelihood that a low-type informed trader may be manipulating the market.

This also highlights the role of informational trade transparency. Figure 5 shows expected profits of the informed trader on a liquidity-adjusted basis for different values of w. The liquidity adjustment removes the non-manipulation related effects of changes in w on liquidity, so that changing w only changes the informational trade transparency of the market. Specifically, larger w's correspond with a market in which it is more difficult (i.e., less likely) for the

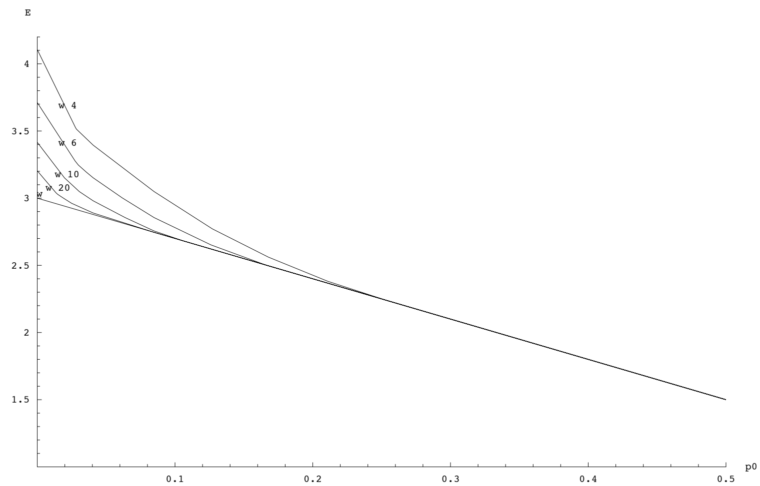

Figure 5: Liquidity-adjusted w-contours of $E(\pi(p_0))$

market maker to infer the direction of informed order flow. The informed trader manipulates less, and profits less, when informational trade transparency decreases. The informed trader benefits when the informational trade transparency is high because this increases the odds that manipulative behavior will be successful in pushing price away from fundamental value. This, in turn, increases the incentive to manipulate, which expands the price region within which the market maker is ambivalent about how to react to informed trade and lowers the price responsiveness to order flow, which ultimately benefits the informed trader.

To summarize: (i) the direct effect of manipulation on informed trader profits is simply to break even, (ii) the benefits of manipulative trade accrue to informed traders who don't manipulate, and (iii) the effects of manipulation are most pronounced in markets with a high degree of informational trade transparency. Thus the benefits to informed traders from manipulation are indirect. The potential of manipulative trade changes the market dynamics to the favor of the informed trader. Specifically, the possibility of manipulation increases market liquidity by making prices more sticky and less responsive to order flow. An informed trader creates (but doesn't profit from) the price stickiness by manipulating when she has a small informational advantage. An informed trader earns excess profits from the increased liquidity by trading with her information when her informational advantage is large. This lends support to Allen and Gale's (1992) claim that trade-based manipulation is difficult to detect and eradicate. Our model suggests that, in general, an informed trader will not earn excess profits and engage in manipulation concurrently. Trade reversals occur when an informed trader has a small informational advantage and could credibly claim to have 'changed their mind' about the asset value. Excess profits occur when an informed trader trades consistently in one direction. If both trade reversals and excess profits are needed to prove manipulation, proof will be difficult.[20]

All else equal, informed traders would like to trade in a market with a high degree of informational trade transparency. This is especially so when an informed trader is expected to have a small informational advantage, but in fact has a large informational advantage. Of course, to some extent one expects informational trade transparency and market liquidity to be inversely related as they are through the joint effect of our w parameter: orders are easier to disaggregate and likely to be less anonymous in markets with low liquidity. Even in such a case the profit curves in Figure 4 indicate that when an informed trader has a large informational advantage she may be willing to sacrifice market depth to gain higher informational trade transparency so long as the market maker believes that the likelihood that an informed trader *might* manipulate is sufficiently high.[21] This is seen, for example, by noticing that the manipulation based profit curve for $w = 4$ would exceed a non-manipulation based (i.e., linear) profit curve for $w = 6$ for prices near 0. In any case, our model suggests that there are circumstances in which an informed trader would wish to make her actions more transparent by "leaking" her trading activity, not breaking up a large order into multiple smaller orders, and the like.

[20] Note, our focus is on understanding the feasibility, dynamics, and profitibility of trade-based manipulation. While our work may be relevant to legal and policy discussions regarding trade-based manipulation, we explicitly are not making any arguments or claims about whether trade-based manipulation is or should be legal or illegal.

[21] Of course the market maker's beliefs can be rational if the informed trader is not expected to have so large an information advantage as she actually does.

4.3 Market Liquidity and Price Efficiency

Figure 6 shows the pre-trade expectation of the post-trade residual variance of the asset's liquidation value (i.e., after the final round of trading but before the liquidation value of the asset is announced). The figure is drawn conditional on the informed trader having recieved a high signal, which creates an asymmetric residual variance profile.[22] Absent manipulation the informed trader trades in the direction of her information each period and the residual variance plots would be parabolas. In our setting there is a constant probability, $2/(2w+1)$, in each round that the informed trader's information will be revealed, independent of initial price. The parabolic profiles therefore simply represent a constant scaling of the initial price variance, which is parabolic owing to the binomial distribution of the informed signal. With a binomial signal uncertainty is a maximum at $p = 1/2$. For very large w it is unlikely that the market maker will perfectly infer the informed trader's information prior to the final trade date and the post-trade residual variance is very close to the ex-ante uncertainty. As w decreases there is an increasing probability that the informed trader's information will be revealed and residual uncertainty profiles are scaled appropriately.

The presence of manipulation changes the residual variance profile in a very significant way. For prices near zero the residual variance plots are not parabolic and, in fact, it is expected that the uncertainty regarding the liquidation value will *increase* over the three rounds of trade. Figure 7 shows a close-up of this price region. This region reflects the change in market dynamics attributable to the manipulative trading strategy. Specifically, an informed trader with a high signal can expect, when price is far from fundamental value, to trade in a more liquid market owing to the effect of manipulation. The price is less responsive to order flow because the market maker is uncertain whether to attribute an informed buy order to manipulation by a low-type informed trader or to profitable trade by a high-type informed trader. In this situation, when price is close to 0, a buy order is very rare: a

[22]Unconditionally, the figure would be symmetric around $p = 0.5$.

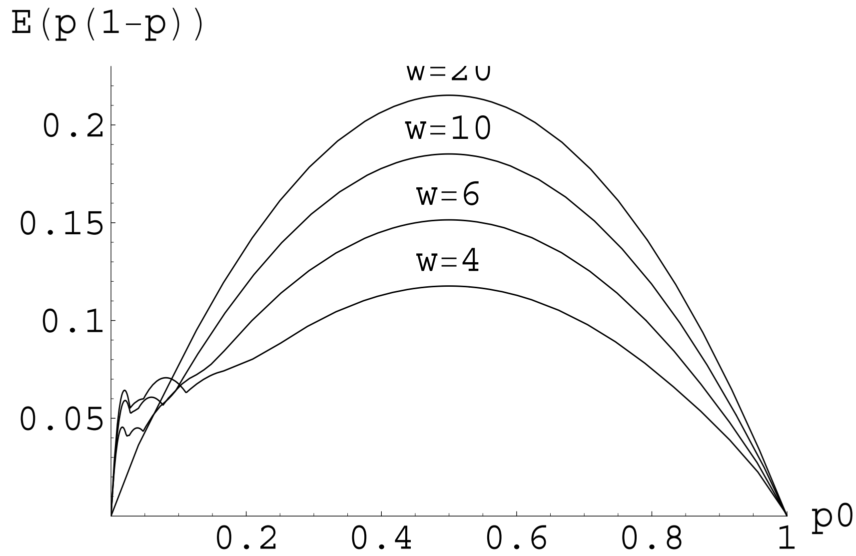

Figure 6: w-contours of $E_0[p_3(1-p_3)]$

low-type informed trader is likely to exist, but she only trades against her information with low probability. A high-type informed trader always trades with her information, but her very existence is rare when price is close to zero. Absent manipulation, price responsiveness could be quite extreme. In particular, an inferred buy order from the informed trader would move price all the way to 1, no matter how close to zero the previous price had been. With manipulation this doesn't happen.

Also note that the expected increase in residual uncertainty over the trading horizon is most pronounced for low values of w. So much so that the effects of manipulation outweigh the effects of increasing liquidity in w. Thus there are significant price regions for which markets with higher levels of noise trade (bigger w) are expected to be more informationally efficient. We provide a new rationale for this result. Naively, one might expect that increasing levels of noise trade would make prices less efficient. Grossman and Stiglitz (1980) argued, on the contrary, that if information production is costly, then prices can become more efficient when noise trade increases because it allows more profitable trading opportunities for informed traders and thereby stimulates information production. Kyle (1985) showed that even absent

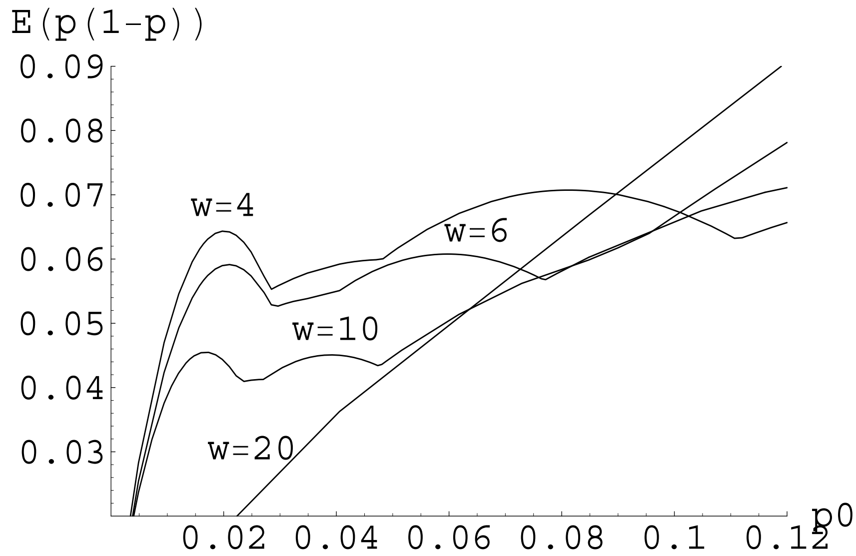

Figure 7: w-contours of $E_0[p_3(1-p_3)]$

costly information production, increasing levels of noise trade needn't impact price efficiency because the intensity of informed trade may increase proportionally. We show that even in the absence of costly information production, increasing noise trade may *increase* price efficiency by diminishing the incentives for manipulative trade. This, again, is why an informed trader may actually prefer to trade in a less liquid market versus a more liquid market, provided concerns about bluffing are larger in the less liquid market.

Lastly, we comment on the scalloping of the residual variance plots. The scalloped shape of the price efficiency curves arises from the discrete changes in market maker beliefs represented by the different price regions in Proposition 1. Within each region there is uncertainty about whether a price change will occur by exiting the region to the right and raising the price, or existing the region to the left and lowering the price. The uncertainty about the direction of the next price update is greatest in the middle of each region, which produces the scalloping.

4.4 Additional Considerations

4.4.1 Information Production

The informed trader in our model is endowed with her information. Here we discuss the interplay between manipulation, price efficiency, and costly information production. Grossman and Stiglitz (1980) showed that if information production is costly, markets must be sufficiently 'noisy' for traders who invest in information to profitably trade on their information. If the market is not 'noisy,' price is a sufficient statistic for private information and uninformed free-riding undermines the incentive to collect costly private information. Market noise is often assumed to come from liquidity-based demand or other supply shocks. Our paper shows that an informed trader can also generate market noise endogenously via trade-based manipulation. All else equal, manipulation increases the expected profit from informed trade and should lead to more information production. Additional information production will offset the negative price efficiency effects of manipulative trade. Therefore, in a setting with costly information production it is not clear whether the net effect of manipulation on expected price efficiency will be positive or negative.

Also recall that the excess profits due to manipulation are convex in the magnitude of the informed trader's informational advantage, as shown in Figure 4. This has several potentially interesting implications. First, this may create increasing returns to scale for information production. Second, if different methods of producing information have different risks with respect to the amount of information produced, the convexity of the expected profits creates a bias toward risk-taking in information production. Last, because there is a higher marginal benefit to generating a lot versus a little information, but because manipulation occurs when an informed trader has a little versus a lot of information, it is possible that a model with endogenous information production may have multiple equilibria or no equilibrium. For example, excess expected profits accruing from a market with manipulation may dictate that an informed trader should collect a lot of information. But if the informed trader does collect a lot of information then her presumption of excess profits is unjustified because no manipulation will occur in equilibrium. However, if the informed trader collects only a little information owing to the lack of excess expected profits, then in equilibrium the informed trader will manipulate and will have been better off having collected more information.

4.4.2 Endogenous Liquidity Trade

The amount of liquidity trade in our model is exogenously specified via w. The effect of endogenizing the liquidity trade is uncertain. On the one hand, endogenous noise trade may have a reinforcing effect on manipulative trade. All else equal, the potential for manipulative trade leads to higher expected informed trader profits. Informed trader profits are paid for with liquidity trader losses. Therefore, if liquidity traders are given some degree of control over when or where they trade, they will choose to avoid times or markets when the potential for manipulative trade are high. As shown above, manipulative trading strategies are more likely to be adopted in illiquid markets because illiquid markets are expected to have a higher degree of informational trade transparency. Therefore, it might be the case that low liquidity and manipulative trading strategies are mutually reinforcing.

On the other hand, manipulative trading strategies are more likely when the expected informational advantage of informed traders is small. All else equal, liquidity traders prefer to trade in a market where the degree of information asymmetry is small. Therefore, if we take the ex ante degree of information asymmetry between informed traders and liquidity traders as exogenous, it may be the case that high liquidity trade and manipulation will be coincident in markets with low information asymmetry while low liquidity and no manipulation will be coincident in markets with high information asymmetry.[23]

[23] See Dow (2004) for a market model with endogenous liquidity trade and multiple equilibria.

4.4.3 Multiple Informed Traders

In our model there is a single informed trader. The existence of multiple informed traders would effect the model significantly. Multiple informed traders would mitigate, if not eliminate, manipulative trade due to free-riding issues. Trading against one's information creates a public good (for the other informed traders), but a personal bad. Informed traders may collectively be better off if they could commit to a trading strategy including manipulative trade, but absent a commitment mechanism, each individual trader may find it in her best interest *not* to engage in manipulation.

It's likely that the correlation among the information of different informed traders may play a significant role. If multiple informed traders have heterogenous information, then it is possible that the informed traders will compete away the common component of their information and then adopt a manipulative trading strategy with respect to the unique component of their information. This intuition is based on the results of Foster and Viswanathan (1996). A more formal treatment of the impact of multiple informed traders is beyond the scope of the current paper and left for future research.

5 Conclusion

This paper derives an equilibrium in which an informed trader engages in trade-based manipulation. We show that a manipulative trade strategy can be profitable even in the absence of mandatory disclosure rules or uncertainty about the presence of an informed trader. We show that manipulative profits are indirect. An informed trader who engages in manipulative trade is likely to be no better off than she would be had she traded strictly in the direction of her information. In contrast, an informed trader who does trade strictly in the direction of her information may earn significant excess profits owing to the presence of the manipulative trading strategy.

In and of itself, manipulative trade undermines price efficiency. However, if the excess profits attributable to adoption of a manipulative trading strategy encourage additional information production, the net effect on market efficiency is unclear. In general, actions and policies that make information-based trades more visible and easier to infer encourage the adoption of manipulative trading strategies. The indirect nature of the profits to manipulative trading strategies make manipulation difficult to detect and eradicate. However, the indirect nature of the profits also makes manipulative trading strategies susceptible to free-rider problems in settings with multiple informed traders. As such, concerns about manipulative trade and it's perceived harms may be more relevant to smaller, less liquid markets with fewer sophisticated participants.

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7 Appendix

7.1 Detailed Strategies and Pricing Rules in Proposition 1 and Corrollary 1

In the first round of trade, the informed trader's trading strategy is the following mixed strategy:

$$X_1(h;p_0) = \begin{cases} -1 & \text{w.p.} \quad \phi_1^h(p_0) \\ 0 & \text{w.p.} \quad \theta_1^h(p_0) \\ +1 & \text{w.p.} \quad 1-\phi_1^h(p_0)-\theta_1^h(p_0) \end{cases}$$

where

$$\phi_1^h(p_0) =$$
$$\begin{cases} 0 & \text{for} \quad 0 \le p_0 \le p_0^{C3} \\ -\left(\frac{1+8\,w+32\,w^2+32\,w^3+48\,w^4+(p_0+2\,p_0\,w)^2\left(1+12\,w^2\right)-2\,p_0\left(1+6\,w+24\,w^2+40\,w^3+48\,w^4\right)}{p_0\left(1+4\,w+16\,w^2+80\,w^3+48\,w^4-p_0\,(1+2\,w)^2\,(1+12\,w^2)\right)}\right) & \text{for} \quad p_0^{C3} \le p_0 \le p_0^{C4} \\ 1-\frac{48w^4+32w^3+48w^2+8w+1}{(12w^2+1)(2w+1)^2}\cdot\frac{1}{p_0} & \text{else} \end{cases}$$

$$\theta_1^h(p_0) = \begin{cases} 0 & \text{for} \quad 0 \le p_0 \le p_0^{C1} \\ \frac{8\,w^2-8\,p_0\,w^2-(-1+p_0)^2\,(-1+2\,w)\,(1+2\,w)^2}{8\,p_0\,w^2-(-1+p_0)\,p_0\,(-1+2\,w)\,(1+2\,w)^2} & \text{for} \quad p_0^{C1} \le p_0 \le p_0^{C2} \\ 1-\frac{4w^2+1}{(2w+1)^2}\cdot\frac{1}{p_0} & \text{for} \quad p_0^{C2} \le p_0 \le p_0^{C3} \\ \frac{-8\,w\left(-1-6\,w-12\,w^2-24\,w^3+p_0\,(1+2\,w)^3\right)}{p_0\,(1+2\,w)^2\left(1+4\,w+16\,w^2+80\,w^3+48\,w^4-p_0\,(1+2\,w)^2\,(1+12\,w^2)\right)} & \text{for} \quad p_0^{C3} \le p_0 \le p_0^{C4} \\ \frac{8w}{12w^2+1}\cdot\frac{1}{p_0} & \text{else} \end{cases}$$

and

$$p_0^{C1} = \frac{-1-2\,w-4\,w^2+8\,w^3}{(-1+2\,w)\,(1+2\,w)^2}$$
$$p_0^{C2} = \frac{1+6\,w+4\,w^2+8\,w^3}{(1+2\,w)^3}$$
$$p_0^{C3} = \frac{1+8\,w+32\,w^2+32\,w^3+48\,w^4}{(1+2\,w)^2\,(1+12\,w^2)}$$
$$p_0^{C4} = \frac{1+6\,w+32\,w^2+144\,w^3+112\,w^4+96\,w^5}{(1+2\,w)^3\,(1+12\,w^2)}$$

$X_1(l;p_0)$ is symmetric. The market maker's pricing rule is:

$$P_1(z_1;p_0) \;=\; \begin{cases} \frac{\phi_1^h(p_0)\cdot p_0}{\phi_1^h(p_0)\cdot p_0+[1-\theta_1^h(p_0)-\phi_1^h(p_0)](1-p_0)} & \text{for} \quad z_1=-w-1 \\ \frac{[\theta_1^h(p_0)+\phi_1^h(p_0)]p_0}{[\theta_1^h(p_0)+\phi_1^h(p_0)]p_0+[1-\phi_1^h(1-p_0)](1-p_0)} & \text{for} \quad z_1=-w \\ p_0 & \text{for} \quad -w+1\le z_1\le w-1 \\ \frac{[1-\phi_1^h(p_0)]p_0}{[1-\phi_1^h(p_0)]p_0+[\theta_1^h(1-p_0)+\phi_1^h(1-p_0)](1-p_0)} & \text{for} \quad z_1=w \\ \frac{[1-\theta_1^h(p_0)-\phi_1^h(p_0)]p_0}{[1-\theta_1^h(p_0)-\phi_1^h(p_0)]p_0+\phi_1^h(1-p_0)\cdot(1-p_0)} & \text{else} \end{cases}$$

In the second round of trade, the informed trader's trading strategy is the following mixed strategy:

$$X_2(h;p_1) \;=\; \begin{cases} 0 & \text{w.p.} \quad \theta_2^h(p_1) \\ +1 & \text{w.p.} \quad 1-\theta_2^h(p_1) \end{cases}$$

where

$$\theta_2^h(p_1) \;=\; \begin{cases} 0 & \text{for} \quad 0\le p_1\le \frac{2w}{2w+1} \\ 1-\frac{2w}{2w+1}\cdot\frac{1}{p_1} & \text{else} \end{cases}$$

$X_2(l;p_1)$ is symmetric. The market maker's pricing rule is:

$$P_2(z_2;p_1) \;=\; \begin{cases} 0 & \text{for} \quad z_2=-w-1 \\ \frac{\theta_2^h(p_1)\cdot p_1}{(1-p_1)+\theta_2^h(p_1)\cdot p_1} & \text{for} \quad z_2=-w \\ p_1 & \text{for} \quad -w+1\le z_2\le w-1 \\ \frac{p_1}{\theta_2^h(1-p_1)\cdot(1-p_1)+p_1} & \text{for} \quad z_2=w \\ 1 & \text{else} \end{cases}$$

In the third and final round of trade, the informed trader's trading strategy is the following pure strategy:

$$X_3(h;p_2) \;=\; 1$$
$$X_3(l;p_2) \;=\; -1$$

and the market maker's pricing rule is:

$$P_3(z_3;p_2) \;=\; \begin{cases} 0 & \text{for} \quad z_3\le -w \\ p_2 & \text{for} \quad -w+1\le z_3\le w-1 \\ 1 & \text{for} \quad z_3\ge w \end{cases}$$

7.2 Proof of Proposition 1

The model is solved by backward induction. The equilibrium is presented and discussed from the perspective of an informed trader with a high signal, $s = h$, without loss of generality. We adopt the following notation. The price in trade round n is p_n. The informed trader's order flow in round n is x_n. The noise trader order flow in round n is u_n. The aggregate order flow is $z_n = x_n + u_n$. The informed trader trade strategy in round n, is:

$$X_n(h; p_{n-1}) \;=\; \begin{cases} -1 & \text{w.p.} \quad \phi_n^h(p_{n-1}) \\ \;\;0 & \text{w.p.} \quad \theta_n^h(p_{n-1}) \\ +1 & \text{w.p.} \quad 1 - \phi_n^h(p_{n-1}) - \theta_1^h(p_{n-1}) \end{cases}$$

7.2.1 Period Three

In the last round of trade before the liquidation value of the risky asset is announced, the informed trader submits his order $x_3 \in \{-1, 0, +1\}$. There are no successive period profits to be considered so the informed trader maximizes expected profit in the current trading round. He is free to choose a mixed strategy over the feasible orders, but trading with his information ($x_3 = +1$) is a (weakly) dominant strategy. The terminal period payoffs $\pi|(p_2, p_1, p_0, s)$ for each pure strategy $x_3 \in \{-1, 0, +1\}$ are given by $p_2 - 1$, 0, and $1 - p_2$ respectively. Since $0 \le p_2 \le 1$, it is trivial to see that $x_3 = +1$ weakly dominates all other strategies. The final round expected profits equal $\frac{2w-1}{2w+1}(1 - p_2)$; there are two possible order flows (out of the 2w+1 order flows possible when $x_3 = 1$) in which the insider's signal is revealed, $z_3 = +w$ amd $z_3 = +w + 1$. In these states, $p_3 = 1$ and the informed trader earns zero profit.

7.2.2 Period Two

Prices

During the second period, the market maker sets prices equal to the expected value of the risky asset, taking as given the strategy of the informed trader. Suppose that observed aggregate order flow were $z_2 = -w-1$. Since the minimum value of the pure noise component is $-w$, the market maker knows that $x_2 = -1$ has been submitted. Only the underlying signal $s \in \{l, h\}$ of the insider is uncertain. Either an insider with $s = h$ (probability = p_1) submitted $x_2 = -1$ which occurs with conditional probability $\phi_2(p_1)$, or an insider with $s = l$ (probability $= 1 - p_1$) submitted $x_2 = -1$ which occurs with conditional probability $1 - \phi_2(1 - p_1) - \theta_2(1 - p_1)$. Applying Bayes Rule, the expected value of the asset given

$z_2 = -w - 1$ is given by:

$$p_2^{++} = p_2|\{z_2 = -w - 1\} = \frac{p_1\phi_2(p_1)}{p_1\phi_2(p_1) + (1 - p_1)[1 - \theta_2(1 - p_1) - \phi_2(1 - p_1)]}$$

All other prices are identically formed, and may be interpreted as the probability that $s = h$ given z_2. The remainder of the prices are given below:

$$\begin{aligned}
p_2^{+} &= p_2|\{z_2 = -w\} = \frac{p_1[\theta_2(p_1) + \phi_2(p_1)]}{p_1[\theta_2(p_1) + \phi_2(p_1)] + (1 - p_1)[1 - \phi_2(1 - p_1)]} \\
p_2^{0} &= p_2|\{-w + 1 \leq z_2 \leq +w - 1\} = p_1 \\
p_2^{-} &= p_2|\{z_2 = +w\} = \frac{p_1[1 - \phi_2(p_1)]}{p_1[1 - \phi_2(p_1)] + (1 - p_1)[\theta_2(1 - p_1) + \phi_2(1 - p_1)]} \\
p_2^{--} &= p_2|\{z_2 = +w + 1\} = \frac{p_1[1 - \phi_2(p_1) - \theta_2(p_1)]}{p_1[1 - \phi_2(p_1) - \theta_2(p_1)] + (1 - p_1)\phi_2(1 - p_1)}
\end{aligned}$$

Expected Profits

The insider's expected profits include those from the second and third rounds of trade. Consider each the strategies $x_2 \in \{+1, 0, -1\}$ in turn. The expected profits from submitting $x_2 = +1$ allow for $-w + 1 \leq z_2 \leq +w + 1$, which eliminate two of the five possible prices. If $u_2 \leq +w - 2$ (which occurs with probability $\frac{2w-1}{2w+1}$), then $p_2^0 = p_1$ as indicated above. Likewise, if $u_2 = +w - 1$ (which occurs with probability $\frac{1}{2w+1}$), $p_2^+ = p_2$. Finally, p_2^{++} is possible if $u_2 = +w$. Period two expected profits, conditional on $x_2 = +1$, written as a function of possible second period prices p_2^0, p_2^+, and p_2^{++} are:

$$\begin{aligned}
\mathrm{E}[\Pi_2|x_2 = +1] &= \frac{2w-1}{2w+1}\left[(1 - p_2^0) + \frac{2w-1}{2w+1}(1 - p_2^0)\right] + \cdots \\
&\quad \frac{1}{2w+1}\left[(1 - p_2^+) + \frac{2w-1}{2w+1}(1 - p_2^+)\right] + \frac{1}{2w+1}\left[(1 - p_2^{++}) + \frac{2w-1}{2w+1}(1 - p_2^{++})\right]
\end{aligned}$$

A similar expression results for $x_2 = 0$. In this case, the most extreme prices (p_2^{++} and p_2^{--}) are precluded:

$$\begin{aligned}
\mathrm{E}[\Pi_2|x_2 = 0] &= \frac{2w-1}{2w+1}\left[0 + \frac{2w-1}{2w+1}(1 - p_2^0)\right] + \cdots \\
&\quad \frac{1}{2w+1}\left[0 + \frac{2w-1}{2w+1}(1 - p_2^+)\right] + \frac{1}{2w+1}\left[0 + \frac{2w-1}{2w+1}(1 - p_2^-)\right]
\end{aligned}$$

Finally, expected profits given $x_2 = -1$ are provided for an insider with signal $s = h$. Now the two highest price regions are impossible, resulting in the following:

$$\mathrm{E}[\Pi_2|x_2 = -1] = \frac{2w-1}{2w+1}\left[(p_2^0 - 1) + \frac{2w-1}{2w+1}(1 - p_2^0)\right] + \cdots$$

$$\frac{1}{2w+1}\left[(p_2^- - 1) + \frac{2w-1}{2w+1}(1-p_2^-)\right] + \frac{1}{2w+1}\left[(p_2^{--} - 1) + \frac{2w-1}{2w+1}(1-p_2^{--})\right]$$

Characterization of Optimal Trading Strategy for Two-Period Trading Game

We conjecture the two-period equilibrium strategy given in Proposition 1, and verify that no profitable deviations exist. For the entire price region, $x_2 = +1$ is submitted with positive probability; the profits from this strategy, therefore, represent the relevant comparison for any potentially profitable deviation. We begin by demonstrating that $x_2 = -1$ is strictly dominated over the possible price range, and can be eliminated from consideration.

Under the conjectured equilibrium, insiders never trade against their information. That is $\phi_2(p_1) = 0$ and $\phi_2(1-p_1) = 0$. There are still five possible prices, but they are greatly simplified. In particular, both price extremes are now fully revealing, i.e. $p_2^{++} = 1$ and $p_2^{--} = 0$. Under the market maker's belief that insiders never trade against their information in the second period, the following condition necessarily holds:

$$\begin{gathered}\tfrac{2w-1}{2w+1}\left[(1-p_1) + \tfrac{2w-1}{2w-1}(1-p_1)\right] + \tfrac{1}{2w+1}\left[\tfrac{(1-p_1)\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1} + \tfrac{2w-1}{2w+1}\tfrac{(1-p_1)\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1}\right] \geq \\ \tfrac{2w-1}{2w+1}\left[(p_1 - 1) + \tfrac{2w-1}{2w+1}(1-p_1)\right] + \tfrac{1}{2w+1}\left[\left(\tfrac{p_1-1}{(1-p_1)+p_1\theta(p_1)}\right) + \tfrac{2w-1}{2w+1}\left(\tfrac{1-p_1}{(1-p_1)+p_1\theta(p_1)}\right)\right] + \\ \tfrac{1}{2w+1}\left[(0-1) + \tfrac{2w-1}{2w+1}(1-0)\right]\end{gathered}$$

The left-hand side, representing the insider's expected two-period profits from trading with his information, is weakly positive for the entire set of possible prices $p \in [0,1]$. The expected profits from trading against one's information are always weakly less than zero for the two-period model, which can never exceed the profits from trading with one's information. The right hand side simplifies to the following, whose value is bounded from above at zero:

$$\frac{2w-1}{2w+1}\left[\frac{-2}{2w+1}(1-p_1)\right] + \frac{1}{2w+1}\left[\frac{-2}{2w+1}\left(\frac{1-p_1}{(1-p_1)+p_1\theta(p_1)}\right)\right] + \frac{1}{2w+1}\left[\frac{-2}{2w+1}\right]$$

By iterated deletion of weakly dominated strategies, we elminate $x_2 = -1$ from further consideration, and restrict our attention to the mixed strategy space spanned by $x_2 \in \{0, +1\}$.

To show that $x_2 = +1$ is strictly dominant for $p_1 \leq \frac{2w}{2w+1}$, we set the expected profits from submitting $x_2 = 0$ and $x_2 = +1$ respectively, simplify, and equate.

$$\begin{gathered}\tfrac{(2w-1)(1-p_1)}{(2w+1)^2}\left[\tfrac{1}{(1-p-1)+p_1\theta(p_1)} + (2w-1) + \tfrac{\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1}\right] = \\ \tfrac{4w(1-p_1)}{(2w+1)^2}\left[2w-1+\tfrac{\theta(1-p_1)}{(1-p_1)\theta(1-p_1)+p_1}\right]\end{gathered}$$

Taking advantage of the symmetric structure of $\theta(p)$, we note that $p_1 \leq \frac{2w}{2w+1} \Rightarrow \theta(p_1) = 0$ necessarily implies that $p_1 \leq \frac{1}{2w+1} \Rightarrow \theta(1-p_1) = 0$. Making this substitution and solving for $\theta(p_1)$ easily results in the expected profits for the two-period game:

$$\mathrm{E}[\Pi_2] = \begin{cases} \frac{4w(2w-1)}{(2w+1)^2}(1-p_1), & \text{if } p_1 \geq \frac{1}{2w+1} \\ \frac{8w^2}{(2w+1)^2}(1-2p_1), & \text{if } p_1 < \frac{1}{2w+1} \end{cases}$$

Thus, in the penultimate round of trade, the informed trader may, depending on the price, choose to not trade rather than trade with his information, but he will never trade against his information.

7.2.3 Period One

We show that the informed insider's strategy is optimal given the market maker's pricing rule and that the pricing rule sets price equal to the expected value of the asset conditional on the aggregate order flow and trade strategy of the informed trader.

Prices

The expressions for prices are identical to those presented in the last section. We present only p_3^{++}, noting that only the time subscripts have been advanced by one position:

$$p_1^{++} = p_1|\{z_1 = -w-1\} = \frac{p_0\phi_1(p_0)}{p_0\phi_1(p_0) + (1-p_0)[1-\theta_1(1-p_0) - \phi_1(1-p_0)]}$$

All other prices are formed identically.

Trading Strategy and Expected Profits

Period one and period two prices are functions of the probability of manipulation (or sitting out), and expected profits, of course, depend on these prices. The functional form of the informed trader mixing probabilities change over the price region $p \in [0,1]$. Consequently, when evaluting the expected payoffs to each strategy, we must consider each region independently. We begin by describing the expected profits conditional on each pure strategy, $\mathrm{E}[\Pi_1|x_1 = +1]$, $\mathrm{E}[\Pi_1|x_1 = 0]$, and $\mathrm{E}[\Pi_1|x_1 = -1]$, and apply these payoffs to each region. The price regions correspond to different mixing probabilitis, although informed traders with different signals manipulate at opposite ends of the price spectrum.

$$\begin{aligned} \mathrm{E}[\Pi_1|x_1 = +1] &= \frac{2w-1}{2w+1}[(1-p_0) + \Pi_2(p_0)] + \frac{1}{2w+1}\left[(1-p_1^+) + \Pi_2(p_1^+)\right] + \cdots \\ &\quad \frac{1}{2w+1}\left[(1-p_1^{++}) + \Pi_2(p_1^{++})\right] \\ \mathrm{E}[\Pi_1|x_1 = 0] &= \frac{2w-1}{2w+1}[(0) + \Pi_2(p_0)] + \frac{1}{2w+1}\left[(0) + \Pi_2(p_1^+)\right] + \frac{1}{2w+1}\left[(0) + \Pi_2(p_1^-)\right] \\ \mathrm{E}[\Pi_1|x_1 = -1] &= \frac{2w-1}{2w+1}[(p_0 - 1) + \Pi_2(p_0)] + \frac{1}{2w+1}\left[(p_1^- - 1) + \Pi_2(p_1^-)\right] + \cdots \end{aligned}$$

$$\frac{1}{2w+1}\left[(p_1^{--}-1)+\Pi_1(p_0^{--})\right]$$

Region 1: $\frac{1+8\,w+32\,w^2+32\,w^3+48\,w^4}{(1+2\,w)^2\,(1+12\,w^2)} \leq p_0 \leq 1$

Given the proposed manipulation strategies in Proposition 1, and the pricing schedule above, the expected profits to each pure strategy are given, after substitution and simplification, as:

$$\mathrm{E}[\Pi_1|x_1=+1]=\mathrm{E}[\Pi_1|x_1=0]=\mathrm{E}[\Pi_1|x_1=-1]=\frac{(1-p_0)\,(-1+2\,w)\left(1+12\,w^2\right)}{(1+2\,w)^3}$$

Facing the same pricing rule, any mixed strategy $\Lambda \in \mathbb{R}_3^+ \equiv \{\gamma_1, \rho_1, 1-\gamma_1-\rho_1; 0 \leq \gamma_1 = Pr(x_1=1) \leq 1 \text{ and } 0 \leq \rho_1 = Pr(x_1=0) \leq 1\}$ over the pure strategy space will also yield the identical payoff given above. Since the informed insider's manipulation schedule $\{\phi_1(p_0), \theta_1(p_0), 1-\phi_1(p_0)-\theta_1(p_0)\} \in \Lambda$, then a rational expectations equilibrium exists at the proposed equilibrium strategy. Thus, the insider will manipulate with the schedule given by Proposition 1, and the market maker will set a price that the insider anticipates. Note that this pricing region encompasses *two* regions where all three pure strategies are utilized with positive probability.

Region 2: $\frac{1+6\,w+4\,w^2+8\,w^3}{(1+2\,w)^3} \leq p_0 \leq \frac{1+8\,w+32\,w^2+32\,w^3+48\,w^4}{(1+2\,w)^2\,(1+12\,w^2)}$

Given the proposed manipulation strategies in Proposition 1, and the pricing schedule above, the expected profits to each pure strategy are given, after substitution and simplification, as:

$$\mathrm{E}[\Pi_1|x_1=+1]=\mathrm{E}[\Pi_1|x_1=0]=\frac{(1-p_0)\,(-1+2\,w)\left(1+12\,w^2\right)}{(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=-1]=\frac{-1+p_0+4\,(-3+2\,p_0)\,w+8\,(-3+p_0)\,w^2+16\,(-5+6\,p_0)\,w^3-48\,(-1+p_0)\,w^4}{4\,w\,(1+2\,w)^3}$$

After some algebra, one can show that for

$$p_0 < \frac{(1-p_0)\,(-1+2\,w)\left(1+12\,w^2\right)}{(1+2\,w)^3}$$

it is the case that:

$$\mathrm{E}[\Pi_1|x_1=+1]=\mathrm{E}[\Pi_1|x_1=0]<\mathrm{E}[\Pi_1|x_1=-1].$$

Since the region of interest excludes this range of prices, $x_1 = -1$ is strictly dominated in this region, and cannot be part of any equilibrium strategy.

For the two remaining undominated pure strategies, their identical payoffs allow us to argue with the same reasoning applied to region 1. Since the market maker anticipates both $x_1 = +1$ and $x_1 = 0$ to be played with positive probability in region 2, the proposed equilibrium strategy represents a rational expectations equilibrium in region 2.

Region 3: $\frac{-1-2\,w-4\,w^2+8\,w^3}{(-1+2\,w)\,(1+2\,w)^2} \le p_0 \le \frac{1+6\,w+4\,w^2+8\,w^3}{(1+2\,w)^3}$

In this region:

$$\mathrm{E}[\Pi_1|x_1 = +1] = \mathrm{E}[\Pi_1|x_1 = 0] = \frac{(1-p_0)\,(-1+2\,w)\left(1+12\,w^2\right)}{(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1 = -1] = -\frac{-1-6\,w+16\,w^2+48\,w^3+272\,w^4-224\,w^5+p_0\left(1+6\,w+8\,w^2+48\,w^3-304\,w^4+224\,w^5\right)}{16\,w^2\,(1+2\,w)^3}$$

For $p_0 > \frac{-1-6\,w+80\,w^3+80\,w^4+160\,w^5}{(1+2\,w)^2\,(-1-2\,w-12\,w^2+40\,w^3)}$ one can show that $\mathrm{E}[\Pi_1|x_1 = -1] > \mathrm{E}[\Pi_1|x_1 = +1]$. However, for $w > 0|w \in \mathbb{R}$:

$$\frac{-1-6\,w+80\,w^3+80\,w^4+160\,w^5}{(1+2\,w)^2\,(-1-2\,w-12\,w^2+40\,w^3)} > \frac{1+6\,w+4\,w^2+8\,w^3}{(1+2\,w)^3},$$

which is strictly outside region 3. Therefore, for the prices within region 3, $\mathrm{E}[\Pi_1|x = -1]$ is strictly dominated, and cannot be part of any equilibrium strategy.

For the two remaining undominated pure strategies, their identical payoffs allow us to argue with the same reasoning applied to region 1 and 2. Since the market maker anticipates both $x_1 = +1$ $x_1 = 0$ with positive probability in region 3, the proposed equilibrium strategy represents a rational expectations equilibrium in region 3.

Region 4: $\frac{8\,w^2}{(-1+2\,w)\,(1+2\,w)^2} \le p_0 \le \frac{-1-2\,w-4\,w^2+8\,w^3}{(-1+2\,w)\,(1+2\,w)^2}$

For this and all remaining regions, the claim is that both manipulation ($x_1 = -1$) and sitting out ($x_1 = 0$) are strictly dominated for an insider facing prices governed by the proposed equilibrium strategies in Proposition 1. In region 4:

$$\mathrm{E}[\Pi_1|x_1 = +1] = -\frac{(-1+p_0)\,(-1+2\,w)\left(1+12\,w^2\right)}{(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1 = 0] = \frac{-4\,w\left(-1+p_0\,(1-2\,w)^2+2\,w-4\,w^2\right)}{(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=-1] = -\frac{1+2\,w+12\,w^2-8\,w^3+p_0\left(1+6\,w-20\,w^2+8\,w^3\right)}{(1+2\,w)^3}$$

It follows that:

$$\mathrm{E}[\Pi_1|x=-1] > E[\Pi_1|x=+1] \iff p_0 > \frac{2\,(w+4\,w^3)}{(-1+2\,w)\,(1+2\,w)^2},$$

which for any $w \in \mathbb{R}$ is impossible within region 4. Therefore, $x_1 = -1$ is strictly dominated by $x_1 = +1$ in this region. Also

$$\mathrm{E}[\Pi_1|x=0] > \mathrm{E}[\Pi_1|x=+1] \iff p_0 > \frac{-1-2\,w-4\,w^2+8\,w^3}{(-1+2\,w)\,(1+2\,w)^2},$$

which by inspection is revealed as the upper border on region 4. Therefore, $x_1 = 0$ is strictly dominated by $x_1 = +1$. Only $x_1 = +1$ survives interated deletion of strictly dominated strategies.

Region 5: $\frac{8\,w^2}{(1+2\,w)^3} \le p_0 \le \frac{8\,w^2}{(-1+2\,w)\,(1+2\,w)^2}$

In this region:

$$\mathrm{E}[\Pi_1|x_1=+1] = -\frac{\left(1+12\,w^2\right)\left[8\,(1-4\,w)\,w^2+p_0\left(-1-2\,w-12\,w^2+40\,w^3\right)\right]}{16\,w^2\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=0] = -\frac{8\,w^2\left(3+8\,w+20\,w^2-32\,w^3\right)+p_0\left(1+10\,w+8\,w^2+16\,w^3-304\,w^4+288\,w^5\right)}{16\,w^2\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=-1] = -\frac{1+2\,w+12\,w^2-8\,w^3+p_0\left(1+6\,w-20\,w^2+8\,w^3\right)}{(1+2\,w)^3}$$

It follows that $\mathrm{E}[\Pi_1|x_1=0] > \mathrm{E}[\Pi_1|x_1=+1] \iff p_0 > \frac{8\,w^2\left(1+6\,w+4\,w^2+8\,w^3\right)}{-1-6\,w-16\,w^2+80\,w^4+96\,w^5}$. However, for $w > 1$, $\frac{8\,w^2\left(1+6\,w+4\,w^2+8\,w^3\right)}{-1-6\,w-16\,w^2+80\,w^4+96\,w^5} > \frac{8\,w^2}{(-1+2\,w)\,(1+2\,w)^2}$, which is the upper bound on region 5. Thus for $p_0 \le \frac{8\,w^2}{(-1+2\,w)\,(1+2\,w)^2}$, $x_1 = 0$ is strictly dominated by $x_1 = +1$.

Similarly, $\mathrm{E}[\Pi_1|x_1=-1] > \mathrm{E}[\Pi_1|x_1=+1] \iff p_0 > \frac{4\,w\left(1+2\,w+8\,w^2\right)}{1-6\,w+12\,w^2+56\,w^3}$, which for all $w > 2$, is strictly greater than the upper bound of region 5.

Region 6: $\frac{1}{1+2\,w} \le p_0 \le \frac{8\,w^2}{(1+2\,w)^3}$

In this region:

$$\mathrm{E}[\Pi_1|x_1=+1] = -\frac{\left(1+12\,w^2\right)\left(p_0-8\,w^2+12\,p_0\,w^2\right)}{4\,w\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=0] = \frac{8\,w\left(-1-4\,w-4\,w^2+8\,w^3\right)+p_0\left(1+12\,w+16\,w^2+80\,w^3-80\,w^4\right)}{4\,w\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=-1] = -\frac{1+2\,w+12\,w^2-8\,w^3+p_0\left(1+6\,w-20\,w^2+8\,w^3\right)}{(1+2\,w)^3}$$

$\mathrm{E}[\Pi_1|x_1 = 0] > \mathrm{E}[\Pi_1|x_1 = +1] \iff p_0 > \frac{4\,w\left(1+5\,w+4\,w^2+4\,w^3\right)}{(1+2\,w)^2\,(1+2\,w+8\,w^2)}$. However, for $w > 2$, $\frac{4\,w\left(1+5\,w+4\,w^2+4\,w^3\right)}{(1+2\,w)^2\,(1+2\,w+8\,w^2)}$ is strictly greater than the upper bound for region 6, $p_0 = \frac{8\,w^2}{(1+2\,w)^3}$. Therefore, $x_1 = 0$ is strictly dominated in this region, and cannot be part of any equilibrium strategy.

$\mathrm{E}[\Pi_1|x_1 = -1] > \mathrm{E}[\Pi_1|x_1 = +1] \iff p_0 > \frac{4\,w\left(1+2\,w+8\,w^2\right)}{1-6\,w+12\,w^2+56\,w^3}$. However, for $w > 0$, $\frac{4\,w\left(1+2\,w+8\,w^2\right)}{1-6\,w+12\,w^2+56\,w^3}$ is strictly greater than the upper bound for region 6. Therefore, $x_1 = -1$ is strictly dominated in this region, and cannot be part of any equilibrium strategy.

Region 7: $\frac{4\,w\left(-1-4\,w+4\,w^2\right)}{(1+2\,w)^2\,(1+12\,w^2)} \leq p_0 \leq \frac{1}{1+2\,w}$

In this region:

$$\mathrm{E}[\Pi_1|x_1=+1] = -\frac{p_0+8\,(1+p)\,w^2-32\,w^3+16\,(-6+13\,p)\,w^4}{4\,w\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=0] = \frac{8\,w\left(-1-6\,w+8\,w^3\right)+p_0\left(1+12\,w+32\,w^2+80\,w^3-144\,w^4\right)}{4\,w\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=-1] = -\frac{1+6\,w+4\,w^2-8\,w^3+p_0\,(1-2\,w)^2\,(1+6\,w)}{(1+2\,w)^3}$$

$\mathrm{E}[\Pi_1|x_1 = 0] > \mathrm{E}[\Pi_1|x_1 = +1] \iff p_0 > \frac{4\,w\left(1+5\,w+4\,w^2+4\,w^3\right)}{(1+2\,w)^2\,(1+2\,w+8\,w^2)}$. However, for $w > 0$, $\frac{4\,w\left(1+5\,w+4\,w^2+4\,w^3\right)}{(1+2\,w)^2\,(1+2\,w+8\,w^2)} > \frac{1}{1+2\,w}$, indicating that $x_1 = 0$ is strictly dominated by $x_1 = +1$ in region 7.

$\mathrm{E}[\Pi_1|x_1 = -1] > \mathrm{E}[\Pi_1|x_1 = +1] \iff p_0 > \frac{4\,w\left(1+2\,w+8\,w^2\right)}{1-6\,w+12\,w^2+56\,w^3}$. However, for $w > 0$, $\frac{4\,w\left(1+2\,w+8\,w^2\right)}{1-6\,w+12\,w^2+56\,w^3} > \frac{1}{1+2\,w}$, indicating that $x_1 = -1$ is strictly dominated by $x_1 = +1$ in region 7.

Region 8: $\frac{8\,w^2\left(-1-8\,w+4\,w^2\right)}{(1+2\,w)^3\,(1+12\,w^2)} \leq p_0 \frac{4\,w\left(-1-4\,w+4\,w^2\right)}{(1+2\,w)^2\,(1+12\,w^2)}$

In this region:

$$\mathrm{E}[\Pi_1|x_1=+1] = -\frac{1+2\,w\left(1+4\,w+4\left(p_0+2\,p\,w+4\,(4+p)\,w^2-2\,(3+4\,p)\,w^3+12\,(-3+8\,p_0)\,w^4\right)\right)}{(-1+2\,w)\,(1+2\,w)^3\,(1+6\,w)}$$

$$\mathrm{E}[\Pi_1|x_1=0] = \frac{8\,w\left(-1-6\,w+8\,w^3\right)+p_0\left(1+12\,w+32\,w^2+80\,w^3-144\,w^4\right)}{4\,w\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=-1] = -\frac{1+6\,w+4\,w^2-8\,w^3+p_0\,(1-2\,w)^2\,(1+6\,w)}{(1+2\,w)^3}$$

$\mathrm{E}[\Pi_1|x_1=0] > \mathrm{E}[\Pi_1|x_1=+1] \iff p_0 > \frac{4\,w\left(-3-16\,w+32\,w^3+48\,w^4\right)}{-1-14\,w-8\,w^2+16\,w^3+304\,w^4+672\,w^5}$. However, for $w > 1$, $\frac{4\,w\left(-3-16\,w+32\,w^3+48\,w^4\right)}{-1-14\,w-8\,w^2+16\,w^3+304\,w^4+672\,w^5}$ is strictly greater than the upper bound for region 8. Therefore, $x_1 = 0$ is strictly dominated in this region.

$\mathrm{E}[\Pi_1|x_1=-1] > \mathrm{E}[\Pi_1|x_1=+1] \iff p_0 > \frac{2\left(-1-6\,w-12\,w^2-32\,w^3+64\,w^4+96\,w^5\right)}{(1+2\,w)^2\,(1+10\,w-52\,w^2+120\,w^3)}$. However, for $w > 1$, $\frac{2\left(-1-6\,w-12\,w^2-32\,w^3+64\,w^4+96\,w^5\right)}{(1+2\,w)^2\,(1+10\,w-52\,w^2+120\,w^3)}$ is strictly greater than the upper bound for region 8. Therefore, $x_1 = -1$ is strictly dominated by $x_1 = +1$ in this region.

Region 9: $0 \le p_0 \le \frac{8\,w^2\left(-1-8\,w+4\,w^2\right)}{(1+2\,w)^3\,(1+12\,w^2)}$

In this region:

$$\mathrm{E}[\Pi_1|x_1=+1] = -\frac{1+6\,w+192\,w^3+112\,w^4-96\,w^5+p_0\,(1-2\,w)^2\left(-1+2\,w+20\,w^2+88\,w^3\right)}{(1+2\,w)^3\,(-1-8\,w+4\,w^2)}$$

$$\mathrm{E}[\Pi_1|x_1=0] = \frac{8\,w\left(-1-6\,w+8\,w^3\right)+p_0\left(1+12\,w+32\,w^2+80\,w^3-144\,w^4\right)}{4\,w\,(1+2\,w)^3}$$

$$\mathrm{E}[\Pi_1|x_1=-1] = -\frac{1+6\,w+4\,w^2-8\,w^3+p_0\,(1-2\,w)^2\,(1+6\,w)}{(1+2\,w)^3}$$

$\mathrm{E}[\Pi_1|x_1=0] > \mathrm{E}[\Pi_1|x_1=+1] \iff p_0 > \frac{4\,w\left(-3-34\,w-88\,w^2-128\,w^3+16\,w^4+32\,w^5\right)}{(1+2\,w)^2\,(-1-20\,w-16\,w^2-112\,w^3+208\,w^4)}$. However, for $w > 3$, this condition cannot be satisfied within the bounds of the region. Therefore, $x_1 = -1$ is strictly dominated by $x_1 = +1$ for region 9.

$\mathrm{E}[\Pi_1|x_1=-1] > \mathrm{E}[\Pi_1|x_1=+1] \iff p_0 > \frac{-\left(1+10\,w+24\,w^2+96\,w^3+16\,w^4-32\,w^5\right)}{8\,w\,(1-4\,w^2)^2}$. However, for $w > 2$, this condition cannot be satisfied within the bounds of the region. Therefore, $x_1 = -1$ is strictly dominated by $x_1 = +1$ for region 9.

Q.E.D.

This completes the proof of the equilibrium strategies proposed in Proposition 1.